**Calibration for hyper-viscoelastic material**

**1. Introduction**

This document proposes a new hyper-Prony calibration process for implementation in the 3D Experience Platform. It is performed by an integrated calibration of the hyperelastic part plus Prony series viscoelastic part simultaneously. It provides a better way to calibrate elastomeric/polymeric materials. The derivation of the new calibration equations is presented. This is followed by a demonstration of the calibration of a Yeoh hyperelastic plus Prony series viscoelastic material model to a series of pseudo experimental results.

**2. Motivation**

Soft rubber-like material is considered as incompressible and isotropic material, whose constitutive relation is hyper-viscoelastic, with linear viscoelasticity described by a Prony series. In order to calibrate this kind of material model in the Abaqus environment (A/CAE ; batch Pre) currently, the constitutive relation has to be divided into two steps. This also means that the user must separate the test data into two parts. In the first step, a purely hyperelastic model regardless of dissipation is considered with the existing models like Neo-Hookean model, Ogden model, Yeoh model, etc. The hyperelastic constants in these models can be calibrated from the experimental data. The second step is viscoelastic calibration, where stress is rate dependent. The problem of this method is that, in most cases, viscous relaxation may occur in the hyperelastic step, elimination of viscoelastic behavior may affect the calibration; and separation of the experimental data into two parts is inappropriate. A new hyper-Prony calibration process is proposed to derive the hyperelastic parameters and the Prony series parameters simultaneously. The new approach is more reasonable since viscous behavior in the hyperelastic region is considered and experimental data can be directly used for the calibration.

**3. Theory**

3.1 Viscoelastic model

For a uniaxial test, the instantaneous nominal stress can be expressed as follows,

|  |  |
| --- | --- |
|  | (1) |

and are the relative moduli of terms . is the number of nonzero terms in and . can be defined by the user.  is the stretch ratio. The integral equation (1) can be analyzed in two separate equations:

|  |  |
| --- | --- |
|  | (2) |

The stress is integrated forward in time. We will assume that the solution is known at time , and we need to construct the solution at time . and are going to be addressed as follows,

|  |  |
| --- | --- |
|  | (3) |

Substituting  into (3) gives,

|  |  |
| --- | --- |
|  | (4) |

The interval of integration can be divided into two intervals for further analysis:

|  |  |
| --- | --- |
|  | (5) |

The integration method for both of the equations in (5) is similar. Here is solved as an example. To integrate the first integral in , it can be converted as follows,

|  |  |
| --- | --- |
|  | (6) |

It is assumed that varies linearly with the reduced time over the increment, which gives,

|  |  |
| --- | --- |
|  | (7) |

Substituting (7) into (6) yields,

|  |  |
| --- | --- |
|  | (8) |

There are three terms in the integral, which are

|  |  |
| --- | --- |
|  |  |

|  |  |
| --- | --- |
|  |  |
|  |  |

Each term in the integration can be further calculated through integration by parts. The details are not specified here; answers are directly given out as follows,

|  |  |
| --- | --- |
|  | (9) |
|  | | |  |

****is instantaneous stress which is going to be calculated through Yeoh model in the next section.

Following the Abaqus Theory manual syntax for gamma, alpha, beta (below eq'n 4.8.2-18)

  

Equations (9) simplify to:





For an equibiaxial test, the instantaneous nominal stress can be expressed as follows,

|  |  |
| --- | --- |
|  | (10) |

and are the relative moduli of terms . is the number of nonzero terms in and . can be defined by the user.  is the stretch ratio. The integration equation can be analyzed in two separate equations, which are as follows,

|  |  |
| --- | --- |
|  | (11) |

The stress is integrated forward in time. We will assume that the solution is known at time , and we need to construct the solution at time . and are going to be addressed as follows,

|  |  |
| --- | --- |
|  | (12) |

Substituting  into (3) gives,

|  |  |
| --- | --- |
|  | (13) |

the interval of integration is divided into two intervals for further analysis:

|  |  |
| --- | --- |
|  | (14) |

The integration method for both of the equations in (5) is similar. Here is solved as an example. To integrate the first integral in, it can be converted as follows,

|  |  |
| --- | --- |
|  | (15) |

It is assumed that varies linearly with the reduced time over the increment, which gives,

|  |  |
| --- | --- |
|  | (16) |

Substituting (16) into (15) yields,

|  |  |
| --- | --- |
|  | (17) |

There are three terms in the integral, which are

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |

Each term in the integration can be further calculated through integration by parts. The details are not specified here, answers are directly given out as follows,

|  |  |
| --- | --- |
|  | (18) |

****is instantaneous stress which is going to be calculated through Yeoh model in the next section.

Following the Abaqus Theory manual syntax for gamma, alpha, beta (below eq'n 4.8.2-18)

  

Equations (18) simplify to:





For a planar test, the instantaneous nominal stress can be expressed as follows,

|  |  |
| --- | --- |
|  | (19) |

and are the relative moduli of terms . is the number of nonzero terms in and .can be defined by the user.  is the stretch ratio. The integration equation can be analyzed in two separate equations, which are as follows,

|  |  |
| --- | --- |
|  | (20) |

The stress is integrated forward in time. We will assume that the solution is known at time , and we need to construct the solution at time .,,and are going to be addressed as follows,

|  |  |
| --- | --- |
|  | (21) |

Substituting  into (21) gives,

|  |  |
| --- | --- |
|  | (22) |

the interval of integration is divided into two intervals for further analysis

|  |  |
| --- | --- |
|  | (23) |

The integration method for both of the equations in (5) is similarHere is solved as an example. To integrate the first integral in , it can be converted as follows,

|  |  |
| --- | --- |
|  | (24) |

It is assumed that varies linearly with the reduced time over the increment, which gives,

|  |  |
| --- | --- |
|  | (25) |

Substituting (25) into (24) yields,

|  |  |
| --- | --- |
|  | (26) |

There are three terms in the integral, which are

|  |  |
| --- | --- |
|  |  |

|  |  |
| --- | --- |
|  |  |
|  |  |

Each term in the integration can be further calculated through integration by parts. The details are not specified here, answers are directly given out as follows,

|  |  |
| --- | --- |
|  | (27) |
|  | | |  |

****and **** are the two components of stress, in the pull direction and in the transverse direction respectively. Both are going to be calculated through Yeoh model in the next section.

Following the Abaqus Theory manual syntax for gamma, alpha, beta (below eq'n 4.8.2-18)

  

Equations (27) simplify to:

|  |  |
| --- | --- |
|  |  |
|  | | |  |

3.2 Yeoh hyperelastic model for incompressible rubbers

The hyperelastic model proposed by Yeoh has a cubic form, which depends on the first strain invariant . The strain energy density for this model is as follows:

|  |  |
| --- | --- |
|  | (28) |

whereare material constants. The Cauchy stress for the incompressible Yeoh model is given by

|  |  |
| --- | --- |
|  | (29) |

where, .

A uniaxial extension test in one direction is taken as an example. The principle stretches are , . Because of the characteristic of incompressibility, we have. Therefore, . The first strain invariant is .

The left Cauchy-Green deformation tensor can be expressed as

|  |  |
| --- | --- |
|  | (30) |

So we have the following equations

|  |  |
| --- | --- |
| **;** | (31) |

For the uniaxial test, ****and are supposed to be zero, which yields , therefore,

|  |  |
| --- | --- |
|  | (32) |

The engineering strain is . The engineering stress can be expressed as

|  |  |
| --- | --- |
|  | (33) |

For an equibiaxial extension test the principle stretches are,. Because of the characteristic of incompressibility, we have. Therefore,. The first strain invariant is.

The left Cauchy-Green deformation tensor can be expressed as

|  |  |
| --- | --- |
|  | (34) |

So we have the following equations

|  |  |
| --- | --- |
| **;** | (35) |

For the equibiaxial test, is supposed to be zero, which yields, therefore,

|  |  |
| --- | --- |
|  | (36) |

The engineering strain is . The engineering stress can be expressed as

|  |  |
| --- | --- |
|  | (37) |

For an planar extension test the principle stretches are, ,. Because of the characteristic of incompressibility, we have. The first strain invariant is.

The left Cauchy-Green deformation tensor can be expressed as

|  |  |
| --- | --- |
|  | (38) |

So we have the following equations

|  |  |
| --- | --- |
| **; ;** | (39) |

For the planar test, is supposed to be zero, which yields, therefore,

|  |  |
| --- | --- |
| **;** | (40) |

The engineering strain is . The engineering stress can be expressed as

|  |  |
| --- | --- |
|  | (41) |

The transverse stress that appears on equations (27) can be expressed as  which results

|  |  |
| --- | --- |
|  | (42) |

**4. Isight Model and Calibration**

A python script is figured out based on the previous models, given 5 series of experimental data. The output of the script is a norm error between the input experimental data and calculated data with specific material parameters. The next move is to utilize the optimization package Isight 5.9 to calibrate the material with the experimental data. One series of the experimental data given is from a fast pull test, the other is from a slow pull test, and the rest three are from stress relaxation test at different time.

The optimization procedure starts with a calibration of hyper-elastic material only with experimental data from the fast pull test. Three parameters are mostly related to hyper-elastic material properties, . The simulation flow in Isight is shown in Fig.1. A document with material properties is put in a data exchanger component named Parameter\_I, a bat file with execution command is inserted in a command component to execute the Python script, another data exchanger component is able to output the norm error. In the optimization component, are taken as three variables with the objective to minimize the norm error. Hooke-Jeeves method is taken as an optimization method. Isight help to find the optimized results for .

Then, a calibration with all the experimental data is conducted with hyper-elastic material parameters and visco-elastic material parameters. The number of Prony series terms will affect the results. Here 5 terms of terms are being. Further research needs be done on the number of terms. In order to make the Isight model more effective, the initial guess from the result of previous optimization is able to help find the parameters faster. The simulation flow is shown in Fig.2. Another four loops are added with specific command component and data exchanger component.

|  |
| --- |
|  |
| Fig. 1 Sim-flow for calibration of hyper-elastic material properties |
|  |
| Fig. 2 Sim-flow for calibration of hyper-viscoelastic material properties |

**5. Conclusion and Suggestions**

5.1 Conclusion

The constitutive relation of hyper-viscoelastic material under the uniaxial test was proposed and derivation was shown, which consists of the viscoelastic Prony series and Yeoh hyperelastic model. The model was calibrated with five series of uniaxial experimental data. The results agree well with all the parameters and errors given.

**Reference**

Chen. T., “Determining a Prony Series for a Viscoelastic Material From Time Varing Strain Data,” NASA/TM-2000-210123 ARL-TR-2206, 2000.

**Appendix**

**Reformulation**

One of the constraints the parameters have to follow is that  and the other can be . Normally, the optimization component in Isight would produce interesting points based on the chosen optimization method. The problem of this normal way is that not all the points are in the constraints, which means the calculation of these points in Isight would be a waste of time. In order to get rid of the useless points and make the Isight model more effective, a reformulation of the parameters can be introduced. Two ways of reformulation are proposed to satisfy two different constraints.

The reformulation for  is as follows,



The reformulation for  is as follows,

Reformulation method is not applied in this model since the current Isight model takes only a few minutes to offer an optimized result. It can be used in the further application.